PROOF OF THE EXISTENCE OF SET OF SOLUTIONS TO THE EQUATIONS OF BEAL'S CONJECTURE AND FERMA'S THEOREM

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1. Abstract. If in literary sources the “Beale conjecture” itself is called the “generalized Fermat’s theorem,” then the equation of the “Beale conjecture” related to number theory is called the generalized Fermat equation in some works. Therefore, the author of the article, having studied these works, in this article offers to the attention of readers many solutions not only to the equation of the “Beale hypothesis”, as well as the equivalent of this hypothesis, but also to the equation of “Fermat’s Last Theorem”.

2. Keywords. Hypothesis, theorem, "Beale conjecture", Beale equation, "Fermat's theorem", Fermat's equation, cube, biquadratic, decomposition of the cube, biquadratic decomposition, doubling of the cube.

3. Introduction.

3.1. About the "Beale hypothesis". The equation of the “Beale hypothesis”, in the fall of 1994, proposed by a mathematician by training and a major American entrepreneur Andrew Beale, for consideration by 50 mathematical journals, specialist scientists and published on the website of the American Mathematical Society [1], as well as in other literary sources [2-10], looks like this:

$$A^x + B^y = C^z, \tag{1}$$

where capital and lowercase letters A, B, C, x, y and z denote positive integers.

Moreover, if positive integers A, B and C must have a common prime factor (CPD), then x, y and z > 2.

And in work [2] and the Wikipedia article "Beal conjecture" it is stated that the "Beal conjecture" is equivalent to the statement that supposedly: equation (1) has no solutions in natural numbers and co-prime numbers A, B and C, if x, y, z ≥ 3.

In order to express this hypothesis and Fermat's hypothesis by means of mathematical logic, corresponding to ISO 31-11 (1992), we introduce the following notation for statements, variables, formulas and equations, according to which:

A, B and C have a common prime divisor – $T = (A, B, C)$;
Beale equation – $E = (A^x + B^y = C^z)$;
set of solutions to the Beale equation – $M = \{A_q^{x_k} + B_q^{y_k} = C_q^{z_k}\}$, где $k \in \mathbb{N} \vee \mathbb{N}^*$, $k = \{0, 1, 2, 3 \ldots\} \vee \{1, 2, 3 \ldots\}$ and $\in \mathbb{N} \vee \mathbb{N}^*, q = \{0, 1, 2, 3 \ldots\} \vee \{1, 2, 3 \ldots\}$.
pairwise coprime numbers – $P = \{1_1, 2_2, , \ldots, \rho_\rho\}$.

Then, if the "Beale conjecture" $A^x + B^y = C^z$ can be formalized by the following formula:

$$\forall x\ \forall y\ \forall z\ E \mid \{A, B, C, x, y, z \in \mathbb{Z}; \mathrm{x, y, z} > 2, T = (A, B, C)\}, \tag{1.1}$$

then Daniel Mauldin's statement, supposedly equivalent to the "Beale conjecture", stated in his article [1] can be formalized by the following formula:

$$\neg \forall x\ \forall y\ \forall z\ E \ \mid \{\mathbb{N} = \{0, 1, 2, 3, 4 \ldots\} \wedge P(1_1, 2_2, , \ldots, \rho_\rho)\} \tag{1.2}$$

or

$$\neg \forall x\ \forall y\ \forall z\ [E = \{A^x + B^y = C^z\}] \wedge M = \{A_q^{x_k} + B_q^{y_k} = C_q^{z_k}\}. \tag{1.3}$$

3.2. About Fermat's hypothesis. Many mathematicians call the hypothesis of amateur mathematician Pierre de Fermat, unproven since 1637, that is, for more than 387 years, "Fermat's Last Theorem." And the British writer Simon Singh even called this theorem "Fermat's Last Theorem." At the same time, mathematicians call this hypothesis a special case of the "Beale conjecture." At the same time, athematicians call this hypothesis a special case of the "Beale conjecture." Therefore, the author of the article, having proven the fact that the "Beale conjecture" has many solutions, decided to dwell on this theorem and prove the existence of solutions and the equation of Fermat's theorem.

In general terms, this theorem was formulated by Pierre Fermat in 1637 in the margins of Diophantus' Arithmetic. The fact is that Fermat made his notes in the margins of the mathematical treatises he read and there he formulated the problems and theorems that came to mind. He wrote down the theorem in question with a note that the ingenious proof of this theorem he found was too long to be placed in the margins of the book:

"On the contrary, it is impossible to decompose a cube into two cubes, a biquadrate into two biquadrates, and in general any power greater than a square into two powers with the same exponent. I have found a truly wonderful proof of this, but the margins of the book are too narrow for it" [5].

The author decided to dwell on these statements of Pierre de Fermat below, in a separate section of this article. But here he wants to note that modern mathematicians have reduced this statement of Fermat to an equation, formulating the following hypothesis that "Fermat's Last Theorem" states that supposedly no three positive integers A, B and C satisfy the equation

$$A^{\mathrm{n}} + B^{\mathrm{n}} = C^{\mathrm{n}} \qquad (2)$$

for any integer value n greater than 2.

This hypothesis can be formalized by the following formula:

$$\neg(\forall n)E(A^n + B^n = C^n) \wedge \left[\mathrm{M} = \left\{\mathcal{C}_q^n = \mathcal{A}_q^n + \mathcal{B}_q^n\right\}\right] | [n \in \mathbb{N}, n\ > 2] \qquad (2.1)$$

4. Materials and Methods.

If expression (1), that is, $A^x + B^y = C^z$ – is nothing other than an equation, then, according to the definitions of equations, it must contain unknown variables and have their roots or solutions. That's how it is. For in the equation $A^x + B^y = C^z$, the unknown variables are not only x, y and z, but also, as shown in this formula (1), such variables, designated as the triple A, B and C. Therefore, the answer to the question: is there a solution to the equation $A^x + B^y = C^z$ or not? - you have to find it by arbitrarily substituting positive integers into this equation, based on the above requirements, that is, by selecting a value, as well as its special case - by exhaustive search, and find the required data that are solutions the equation.

In this case, equation (1) can be solved in three ways, alternately, considering unknown variables: 1) C and z in C^z, when their values are found from the equation $C^z = A^x + B^y$; 2) A and x in A^x, when their values are found from the equation $A^x = C^z - B^y$ and 3) B and y in B^y, when their values are found from $B^y = C^z - A^x$. And to do this, instead of the corresponding unknown variables, it is necessary to substitute various positive integers based on the method of selecting values.

The author of the article in his book "Economic Logic" [4] introduced a notation in the form $\int$ for substituting various numbers into formulas. Therefore, in this article, in those formulas where there was a need to substitute instead of unknown variables such as A^x, B^y and C^z from equation (1) such power numbers designated as: $A_q^{x_k}$, $B_q^{y_k}$and $C_q^{z_k}$, where $k = \{0, 1, 2, 3 \ldots\} \vee \{1, 2, 3 \ldots\}$ and $q = \{0, 1, 2, 3 \ldots\} \vee \{1, 2, 3 \ldots\}$, then , assuming the known values of the variables: A^x, B^y, A, B, x and y, if the values of the variables C^z, C and z are found, then such an algebraic operation can be written in general form, in the following way:

$$\int_{A^x,\, B^y}^{A_q^{x_k}, B_q^{y_k}} A^x + B^y = C^z \Rightarrow \{A_q^{x_k} + B_q^{y_k} = C_q^{z_k} |\, C^z = C_q^{z_k}, C = C_q, z = z_k\}. \qquad (3)$$

And for other cases, formula (3) was modified; if the values of the variables A^x, A and x are found, then such an algebraic operation can be written in general form as follows:

$$\int_{C^z,\, B^y}^{C_q^{z_k}, B_q^{y_k}} C^z - B^y = A^x \Rightarrow \{C_q^{z_k} - B_q^{y_k} = A_q^{x_k} |\, A^x = A_q^{x_k}, A = A_q, x = x_k\}. \qquad (4)$$

If the values of the variables B^y, B and y are sought, then such an algebraic operation in general form can be written as follows:

$$\int_{C^z, A^x}^{C_q^{z_k}, A_q^{x_k}} C^z - A^x = B^y \Rightarrow \{C_q^{z_k} - A_q^{x_k} = B_q^{y_k} \mid B^y = B_q^{y_k}, B = B_q, y = y_k\}. \quad (5)$$

Now we can move on to the results of the research by the author of this article.

5. Results

5.1. Proofs of the existence of multiple solutions to the Beale conjecture equation

The data obtained in the above ways were placed in the table of the Appendix. However, the author's research showed that a single positive integer, denoted by A^x or B^y on the left side of equation (1), could be written by a set of power numbers. As a result of this state of affairs, the solution of equation (1), i.e., $A^x + B^y = C^z$, can be written by a variety of numerical equations that make up power numbers, as shown in the next part of the table in the Appendix, where only 4 examples out of 47 solutions of Beale's equation are shown under numbers No 4, 8, 25, and 38.

Positive integers found by solving the Beale equation Brute force

№	$A^x + B^y = C^z$	CPD A, B и C	A^x	B^y	C^z
4	$2^{16} + 2^{16} = 2^{17}$	**2**	**65536**	**65536**	**131072**
8	$4^{10} + 16^5 = 8^7$	**2**	**1048576**	**1048576**	**2097152**
22	$32^8 + 16^{10} = 2^{41}$	**2**	**1099511627776**	**1099511627776**	**2199023255552**
38	$27^6 + 1458^3 = 81^5$	**3**	**387420489**	**3099363912**	**3486784401**

At the same time, if we turn to the table of the Appendix itself, we can see that if the positive integer 65536 under No 4 corresponds to only 3 equations, the number 1048576 under No 8 corresponds to only 12, the number 1099511627776 under No 22 corresponds to only 12, then the number 387420489 under No 38 corresponds to 5 equations.

In order to describe this circumstance, if we denote equation (1) by $Y = (A^x + B^y = C^z)$ and $X = \left(A_q^{x_k} + B_q^{y_k} = C_q^{z_k}\right)$, we can write it as follows: $f: Y \longrightarrow X_g^r$, where $r = k_1, k_2, \dots k_a$ and $g = b_1, b_2, \dots b_b$. And we can say that this is an expression to say that the Beale equation has many images.

The author of the article studied many literary sources, some of which contain not only not very convincing counterexamples [1-3, 5-13], which some mathematicians and authors of Wikipedia articles call erroneous, but also analytical formulas for solving the Beale equation.

Theorem-1. For positive integers $\boldsymbol{A}$, $\boldsymbol{B}$ and $\boldsymbol{C}$, which have a common prime divisor, at x, y, z > 2, the equation $\boldsymbol{A^x + B^y = C^z}$ has many solutions.

This theorem can be formalized as follows:

$$\forall x\, \forall y\, \forall z\, [E = \{A^x + B^y = C^z\}] \wedge M = \left\{A_q^{x_k} + B_q^{y_k} = C_q^{z_k}\right\} \tag{1.4}$$

Proofs of Theorem-1. Taking into account that in the table of the Appendix the author of the article gave 47 solutions of Beal's equation, and at this point in this article he gives the following formula for solving this equation, abbreviated than the terms of equation (3), proving the validity of theorem-1, which has the following form:

$$\int_{A_q^{x_k}, B_q^{y_k}}^{10^8,\, 2000^3} A_q^{x_k} + B_q^{y_k} = C_q^{z_k} \Rightarrow \mathbf{10^8 + 2000^3 = 300^4} \blacksquare \tag{6}$$

At the same time, it is easy to see that the common prime divisor of the numbers A, B and C in expression (6) is 2. $\frac{A=10}{2} = 5$; $\frac{B=2000}{2} = 1000$ and $\frac{C=300}{2} = 150$. But these data are not included in the table of the Appendix.

In addition, before proposing a general analytical formula for determining the set of values of Beal's equation $A^x + B^y = C^z$, three more equations with numerical values in an abbreviated form than those written in formula (6) are presented for the consideration of the reader in order to place them on this page. And so, here are these formulas:

$$\mathbf{10^{20} + 20000000^3 = 300000^4};$$

$$\mathbf{10^{32} + 200000000000^3 = 300000000^4;}$$

$$\mathbf{10^{44} + 2000000000000000^3 = 300000000000^4}.$$

Now it is possible to present for the consideration of readers a general analytical formula for determining the values of variables in the Beale equation (1). This analytical formula can be derived by writing down the above solutions in the following ordered and abbreviated form:

$$\mathbf{10_1^{0+8_0} + 10^0 \cdot 2000_1^{3_0} = 10^0 \cdot 300_1^{4_0}};$$

$$\mathbf{10_2^{12+8_1} + 10^{12} \cdot 2000_1^{3_1} = 10^{12} \cdot 300_1^{4_1}};$$

$$\mathbf{10_3^{24+8_2} + 10^{24} \cdot 2000_1^{3_2} = 10^{24} \cdot 300_1^{4_2}};$$

....;

$$\mathbf{10_q^{12k+8_k} + 10^{12k} \cdot 2000_1^{3_k} = 10^{12k} \cdot 300_1^{4_k}}, \qquad (7)$$

where is $q = \{1, 2, 3, \ldots\};\ k = \{0, 1, 2, 3, \ldots\}$.

Another formula can be presented to the readers, which is as follows:

$$\int_{A_q^{x_k}, B_q^{y_k}}^{2^3,\, 2^3} A_q^{x_k} + B_q^{y_k} = C_q^{z_k} \Rightarrow \mathbf{2^3 + 2^3 = 2^4}, \qquad (8)$$

where q and k have the same values as above.

In order to derive a general analytical formula for the solution of the Beale equation, the following solutions of this equation are obtained from the use of formula (8) and written in the usual form:

$$\mathbf{20000^3 + 20000^3 = 2000^4};$$

$$\mathbf{200000000^3 + 200000000^3 = 2000000^4};$$

$$\mathbf{2000000000000^3 + 2000000000000^3 = 2000000000^4};$$

$$\mathbf{20000000000000000^3 + 20000000000000000^3 = 2000000000000^4},$$

can be written as follows:

$$(\mathbf{2_1 \cdot 10^0})^3 + (\mathbf{2_1 \cdot 10^0})^3 = (\mathbf{2_1 \cdot 10^0})^4$$
$$(\mathbf{2_2 \cdot 10^4})^3 + (\mathbf{2_2 \cdot 10^4})^3 = (\mathbf{2_2 \cdot 10^3})^4$$
$$(\mathbf{2_3 \cdot 10^8})^3 + (\mathbf{2_3 \cdot 10^8})^3 = (\mathbf{2_3 \cdot 10^6})^4$$
$$(\mathbf{2_4 \cdot 10^{12}})^3 + (\mathbf{2_4 \cdot 10^{12}})^3 = (\mathbf{2_4 \cdot 10^9})^4$$
$$(\mathbf{2_5 \cdot 10^{16}})^3 + (\mathbf{2_5 \cdot 10^{16}})^3 = (\mathbf{2_5 \cdot 10^{12}})^4$$

....

$$\left(\mathbf{2_q \cdot 10^{4k}}\right)^3 + \left(\mathbf{2_q \cdot 10^{4k}}\right)^3 = \left(\mathbf{2_q \cdot 10^{3k}}\right)^4. \qquad (9)$$

Thus, having obtained another general analytical formula (9) for the solutions of Beal's equation (1), we can now proceed to proofs of the next theorem.

Theorem-2. The equation of the "Beale conjecture" mathbit $\boldsymbol{A^x + B^y = C^z}$ has many solutions in natural and pairwise mutually prime numbers *A, B,* and $\boldsymbol{C}$, provided that *x, y, z, ≥ 3*.

This theorem can be formalized as follows:

$$\forall x \; \forall y \; \forall z \; E \wedge \left[M = \left\{A_q^{x_k} \; + \; B_q^{y_k} = C_q^{z_k}\right\} \mid P = \left\{1_1, 2_2, , \ldots, \rho_\rho\right\}\right], \qquad (1.5)$$

where is $E \; = A^x + B^y = C^z$.

Proofs of Theorem-2. Below is an equation and its solution, based on formulas (3-9), proving theorem-2 and the fallacy of Daniel Mauldin's statements [2]. This equation is also abbreviated from equation (3) to fit it on this page:

$$\int_{A_q^{x_k}, B_q^{y_k}}^{80^{21}, 87^{20}} A_q^{x_k} \; + \; B_q^{y_k} = C_q^{z_k} \; \Rightarrow \mathbf{80^{21} \; + \; 87^{20} = 21429198478298^3} \; \blacksquare \qquad (10)$$

In addition, the following 3 solutions of Beale's equation (1) are proposed in an abbreviated form than it was written even in formula (10), for the above reason:

$$\mathbf{50^{25} + 57^{25} = 434170501580651^3};$$

$$\mathbf{55^{26} + 67^{21} = 1210997724514610^3};$$

$$\mathbf{29^{33} + 67^{27} = 28000948240770100^3}.$$

In these formulas, the values of the variables A, B and C are not only natural numbers, but also pairs of mutually prime numbers. Their status as pairs of mutually prime numbers was established by finding the GCF and LCM of these numbers on the calculator of the skysmart.ru website and checked by other existing methods and methods to establish their status.

5.2. Proofs of the existence of solutions to the equation "Fermat's Last Theorem"

The author of the article notes the fact that the absolute majority of mathematicians claim that Fermat's Last Theorem was allegedly proved by the English mathematician Sir Andrew John Wiles [10]. However, the author of this article, having familiarized himself with the proofs of this mathematician, came to the conclusion that he was unable to prove Fermat's Last Theorem. Therefore, he decided to prove the fallacy of this hypothesis on the basis of his own theorems.

Theorem-3. The equation of Fermat's theorem mathbit $\boldsymbol{A^{\mathrm{n}} + B^{\mathrm{n}} = C^{\mathrm{n}}}$ for any integer *n* greater than 2 has many solutions.

This theorem can be expressed as follows:

$$\forall n\ [E = \{A^n + B^n = C^n\}] \wedge [\mathrm{M} = \{\mathcal{C}_k^n = \mathcal{A}_k^n + \mathcal{B}_k^n\}] \mid [n \in \mathbb{N}, n > 2] \quad (2.2)$$

Proofs of Theorem-3. Formula (2.2) means that for equation (2) there are multiple solutions at $n \geq 3$. And the author of the article found one of such solutions at the value of $n = 3$, when the equation $A^{\mathrm{n}} + B^{\mathrm{n}} = C^{\mathrm{n}}$ could be written as $\mathcal{C}_k^n = \mathcal{A}_k^n + \mathcal{B}_k^n$. Therefore, on the basis of finding and substituting the corresponding positive integers, and denoting that: $A^3 = \mathcal{A}_1^3 = 670\,000\,000\,000\,000^3$,

$B^3 = \mathcal{B}_1^3 = 580\,000\,000\,000\,000^3$, $C^3 = \mathcal{C}_1^3 = 791\,511\,819\,514\,367^3$, this equation can be written as:

$$\int_n^3 A^n + B^n = C^n \Rightarrow A^3 + B^3 = C^3 \wedge \mathcal{A}_1^3 + \mathcal{B}_1^3 = \mathcal{C}_1^3. \quad (11)$$

And it follows that:

$$\int_n^3 A^n + B^n = C^n \Rightarrow A = \sqrt[3]{\mathcal{A}_1^3} = \mathcal{A}_1, B = \sqrt[3]{\mathcal{B}_1^3} = \mathcal{B}_1, C = \sqrt[3]{\mathcal{C}_1^3} = \mathcal{C}_1. \quad (12)$$

Thus, equation (2) can be finally written as the following equation by substituting their corresponding numerical values in the place of the notation $\mathcal{A}_1^3 + \mathcal{B}_1^3 = \mathcal{C}_1^3$:

$$(\mathbf{2000} \cdot \mathbf{10^{12}})^{\mathbf{3}} + (\mathbf{1000} \cdot \mathbf{10^{12}})^{\mathbf{3}} = \mathbf{2080083823051900^3} \blacksquare \quad (13)$$

5.3. A bisquare can be decomposed into two bisquares

Now we can return to the analysis of Fermat's statement or hypothesis about "*the impossibility of decomposing a cube into two cubes, a bisquare into two bisquares, and in general no power greater than a square into two powers with the same exponent*". First, let's consider the issue of bisquare decomposition.

Theorem-4. It is possible to decompose a bisquare into two bisquares, that is, one positive integer of degree 4 can be decomposed into two positive integers of degree 4.

This theorem can be formalized in the form of:

$$\forall A\ \forall B\ \forall C\ [E = \{C^n = A^n + B^n\}] \wedge \left[\mathrm{M} = \left\{\mathcal{C}_q^n = \mathcal{A}_q^n + \mathcal{B}_q^n\right\} | n = 2^2 = 4\right] \quad (2.3)$$

Proofs of Theorem-4. In order to prove this theorem at a value of $= 2^2$, the equation $A^{\mathrm{n}} + B^{\mathrm{n}} = C^{\mathrm{n}}$ can be written as $C^{2^2} = A^{2^2} + B^{2^2}$. Therefore, on the basis of finding and substituting the corresponding positive integers in their places, as well as making the notation: $C^{2^2} = \mathcal{C}_2^4 = 544808628412873^4$, $A^{2^2} = \mathcal{A}_2^4 = 400000000000000^4$, $B^{2^2} = \mathcal{B}_2^4 = 500000000000000^4$, this biquadratic equation can be written as follows:

$$\int_n^4 C^n = A^n + B^n \Rightarrow C^4 = A^4 + B^4 \wedge \mathcal{C}_2^4 = \mathcal{A}_2^4 + \mathcal{B}_2^4. \quad (14)$$

And it follows that:

$$\int_n^4 C^n = A^n + B^n \Rightarrow C = \sqrt[4]{\mathcal{C}_2^4} = \mathcal{C}_2, A = \sqrt[4]{\mathcal{A}_2^4} = \mathcal{A}_2, B = \sqrt[4]{\mathcal{B}_2^4} = \mathcal{B}_2. \quad (15)$$

Thus, substituting their respective numerical values for mathcal$\mathcal{C}_2^4$, $\mathcal{A}_2^4$ and $\mathcal{B}_2^4$, the equation $\mathcal{C}_2^4 = \mathcal{A}_2^4 + \mathcal{B}_2^4$, which decomposes one bisquare into two bisquares, can be finally written as the following expression:

$$\boldsymbol{\mathcal{C}_2^4 = 544808628412873_2^4} = (\mathbf{400 \cdot 10^{12}})_2^4 + (\mathbf{500 \cdot 10^{12}})_2^4. \quad (16)$$

As an example, I give three more equations that decompose one bisquare into two bisquares, that is, by substituting their corresponding numerical values in the place of $\mathcal{C}_k^4$, $\mathcal{A}_k^4$ and $\mathcal{B}_k^4$, the equation mathcal$\mathcal{C}_k^4 = \mathcal{A}_k^4 + \mathcal{B}_k^4$ for $k = 3$, 4 and 5 can be written as follows:

$$\mathcal{C}_3^4 = 779762530865908_3^4 = (600 \cdot 10^{12})_3^4 + (700 \cdot 10^{12})_3^4; \quad (17)$$

$$\mathcal{C}_4^4 = 1016035170368110_4^4 = (800 \cdot 10^{12})_4^4 + (900 \cdot 10^{12})_4^4; \quad (18)$$

$$\mathcal{C}_5^4 = 1252894729119000_5^4 = (1000 \cdot 10^{12})_5^4 + (1100 \cdot 10^{12})_5^4. \quad (19)$$

5.4. On the Possibility of Decomposing a Cube into Two Cubes

Theorem-5. Any cube can be decomposed into two cubes.

By denoting by C^3 the decomposable cube, and by A^3 and B^3 the volumes of cubes obtained after the decomposition of the cube C^3, this theorem can be written as:

$$\forall A\ \forall B\ \forall C[E = \{C^n = \mathrm{A}^n + B^n\}]\ \wedge\ \left[\mathrm{M} = \left\{\mathcal{C}_q^n = \mathcal{A}_q^n + \mathcal{B}_q^n\right\} | n = 3\right]. \qquad (2.4)$$

Proofs of Theorem-5. Formula (2.4) means that the decomposition of a cube into two cubes has multiple solutions at $n = 3$. And one of these solutions to the general equation $C^{\mathrm{n}} = A^{\mathrm{n}} + B^{\mathrm{n}}$with a value of $n = 3$ can be written as the equation $\mathcal{C}_q^3 = \mathcal{A}_q^3 + \mathcal{B}_q^3$.

Therefore, by finding and substituting the corresponding positive integers in their places, and denoting the existing variables in such a way that: $C^3 = \mathcal{C}_6^3 = 208008382305190^3$, $A^3 = \mathcal{A}_6^3 = 100000000000000^3$, $B^3 = \mathcal{B}_6^3 = 200000000000000^3$, this equation can be written as:

$$\int_n^3 C^n = A^n + B^n \Rightarrow\ C^3 = A^3 + B^3 \vee \mathcal{C}_6^3 = \mathcal{A}_6^3 + \mathcal{B}_6^3. \qquad (20)$$

And it follows that:

$$\int_n^3 C^3 = A^3 + B^3 \Rightarrow A = \sqrt[3]{\mathcal{A}_6^3} = \mathcal{A}_6, B = \sqrt[3]{\mathcal{B}_6^3} = \mathcal{B}_6, C = \sqrt[3]{\mathcal{C}_6^3} = \mathcal{C}_6. \qquad (21)$$

Thus, equation (2), i.e. $C^3 = A^3 + B^3$, can be finally written as the following equation by substituting their respective numerical values for the notations $\mathcal{C}_6^3, \mathcal{A}_6^3$ and $\mathcal{B}_6^3$:

$$\boldsymbol{\mathcal{C}_7^3 = 449794144527539_7^3 = (300 \cdot 10^{12})_7^3 + (400 \cdot 10^{12})_7^3}; \qquad (23)$$

$$\boldsymbol{\mathcal{C}_8^3 = 698636802781807_8^3 = (500 \cdot 10^{12})_8^3 + (600 \cdot 10^{12})_8^3}; \qquad (24)$$

$$\boldsymbol{\mathcal{C}_9^3 = 949121995802932_9^3 = (700 \cdot 10^{12})_9^3 + (800 \cdot 10^{12})_9^3}. \qquad (25)$$

5.5. A cube can be divided into two cubes by doubling it

There is another very important and accurate way to prove the possibility of decomposing a cube into two cubes is associated with the following ancient legend.

According to this ancient legend [11], one day an epidemic of plague broke out on the island of Delos. The inhabitants of the island turned to the Delphic oracle, and he said that it was necessary to double the altar of the sanctuary, which had the shape of a cube. The inhabitants of Delos built a second cube and put it on the first, but the epidemic did not stop. After a second appeal, the oracle explained that the doubled altar should be a single cube.

Since then, the Delhi problem has been studied by the best mathematicians of the ancient world and several solutions have been proposed. However, no one was able to carry out such a construction using only a compass and a ruler, so a general conviction gradually developed that such a problem was unsolvable.

Perhaps that is why Aristotle wrote in the fourth century BC: "By means of geometry it is impossible to prove that... two cubes make one cube" [10-12].

The author of the article believes that Fermat exhausted his hypothesis about the impossibility of decomposing a cube into two cubes from the above-quoted statement of Aristotle. However, the issue of doubling the cube was solved in ancient times, more precisely in the IV century [11-13]. However, the Wikipedia article "Doubling the cube" states that $\sqrt[3]{2}$ is not a power of two, therefore, based on the above, "$\sqrt[3]{2}$ is not the coordinate of a constructible point, and thus a line segment of $\sqrt[3]{2}$ cannot be constructed, and the cube cannot be doubled".

However, when it comes to an irrational number denoted by the letter Pi, that is, $\pi = 3{,}1415926535\ldots$ Such conversations are not conducted. Although it is, I would say, an irrational number and does not have such a remarkable property as the analogous number $\sqrt[3]{2}$ associated with the transformation of this number from the irrational to the positive integer 2, in the image of the volume of the cube $\left(\sqrt[3]{2}\right)^3$. That is why in the works of ancient and later authors everything comes down to the construction of a segment with a length of $\sqrt[3]{2}$.

And modern authors and their co-authors in the above-mentioned Wikipedia article "Doubling the cube" with reference to Hippocrates of Chios write an expression of the following form: $r = a \cdot \sqrt[3]{2}$, where a is the length of the edge of the cube. Then, denoting the volume of the doubled cube by C^3, we can write that:

$$\mathrm{C}^3 = a^3 \cdot \left(\sqrt[3]{2}\right)^3 = 2. \tag{26}$$

At the same time, it should be noted that the French mathematician Pierre Wenzel in his article [14], published in 1837, proved that it was impossible to solve such geometric problems with the help of a compass and a ruler. However, a lot of time has passed since then, and now, at the beginning of the 21st century, it is safe to say that modern specialists, with the help of virtual tools such as a compass and a ruler, can build not only a segment with a beam $\sqrt[3]{2}$, but also solve the problem of building the doublest cube in various drawing programs, such as AutoCAD, nanoCAD, A9CAD, including programs, working online, as on the tinkercad.com website. Moreover, they can animate their builds as proof of their decisions, as is done on WIKIMEDIA COMMONS.

So, in order to show and prove the decomposition of such a cube into cubes, we will also introduce the following notations: $\mathrm{C}^3 = C_a^3$ and $\mathcal{A}_a^3 = \mathcal{B}_a^3 = 1^3$, where $\mathcal{A}_a^3$ and $\mathcal{B}_a^3$ – are the volumes of cubes that make up the volume of the doubled cube denoted by $C_a^3 = \mathcal{A}_a^3 + \mathcal{B}_a^3$.

The question of doubling a cube, or rather doubling the volume of such a cube, at $a = 1$, $C_1^3 = 1^3 \cdot \left(\sqrt[3]{2}\right)^3$ and unit volumes of cubes, when: $\mathcal{A}_1^3 = 1^3$ и $\mathcal{B}_1^3 = 1^3$, can be determined from Eq. $C_1^3 = \mathcal{A}_1^3 + \mathcal{B}_1^3$, as follows:

$$C_1^3 = 2a^3 = a^3 \cdot \left(\sqrt[3]{2}\right)^3 = 2 \cdot 1^3 \vee C_2^3 = 2 \cdot 1^3 = 1 \cdot 1^3 + 1 \cdot 1^3. \tag{27}$$

I suppose that the authors who tried to solve Fermat's equation did not pay any attention to the question of doubling the cube because in this question the basis for solving the problem is, first, not a positive integer, but an irrational number $\sqrt[3]{2}$. And, secondly, in the question of doubling the cube was the problem of doubling the volume of one or existing cube, that is, the inverse problem, the decomposition of one cube into two cubes.

Based on an in-depth study of this issue, the following theorem can be formulated.

Theorem-6. **One can create infinitely many doubled cubes. Any such cube can be decomposed into two cubes, i.e. doubles it at the expense of other two cubes of smaller size.**

This statement can be formalized in the form of the following formula:

$$\forall a \{C_a^n = \mathcal{A}_a^n + \mathcal{B}_a^n\} \wedge \left\{C_a^3 = a^3 \cdot \left(\sqrt[3]{2}\right)^3 = 2a^3\right\} \mid [a = 1, 2, \ldots, \lambda], [n = 3]. \quad (2.5)$$

Proof of Theorem-6. In order to prove the validity of Theorem-6, formalized in formula (2.5), taking into account the fact that the length of the edge of the cube under consideration has already been taken to be equal to $a = 1$, Now, if instead of the edge length of the cube under consideration we substitute its values one by one $a = 1, 2, 3, \ldots, \lambda$, then by slightly reducing expression (27) we can obtain the following results and a generalized analytical formula in the form of the final expression (28):

$$\boldsymbol{C_a^3 = a^3 \cdot \left(\sqrt[3]{2}\right)^3 \Rightarrow C_1^3 = 2^1 \cdot 1^3 = 1 \cdot 1^3 + 1 \cdot 1^3};$$

$$\boldsymbol{C_a^3 = a^3 \cdot \left(\sqrt[3]{2}\right)^3 \Rightarrow C_2^3 = 2^3 \cdot 2 \cdot 1^3 = 8 \cdot 2 \cdot 1^3 = 16 \cdot 1^3 = 8 \cdot 1^3 + 8 \cdot 1^3};$$

$$\boldsymbol{C_a^3 = a^3 \cdot \left(\sqrt[3]{2}\right)^3 \Rightarrow C_3^3 = 3^3 \cdot 2 \cdot 1^3 = 27 \cdot 2 \cdot 1^3 = 54 \cdot 1^3 = 27 \cdot 1^3 + 27 \cdot 1^3};$$

$$\boldsymbol{C_a^3 = a^3 \cdot \left(\sqrt[3]{2}\right)^3 \Rightarrow C_4^3 = 4^3 \cdot 2 \cdot 1^3 = 64 \cdot 2 \cdot 1^3 = 128 \cdot 1^3 = 64 \cdot 1^3 + 64 \cdot 1^3};$$

..;

$$\boldsymbol{C_a^3 = a^3 \cdot \left(\sqrt[3]{2}\right)^3 \Rightarrow C_{10}^3 = 10^3 \cdot 2 = 10^3 \cdot 2 \cdot 1^3 = 10^3 \cdot 1^3 + 10^3 \cdot 1^3};$$

..;

$$\boldsymbol{C_a^3 = a^3 \cdot \left(\sqrt[3]{2}\right)^3 \Rightarrow C_a^3 = 2 \cdot \lambda^3 \cdot 1^3 = 1 \cdot \lambda^3 \cdot 1^3 + 1 \cdot \lambda^3 \cdot 1^3} \blacksquare \qquad \textbf{(28)}$$

6. Conclusions.

1- The fact that the equation of Beale's Hypothesis has many solutions is proved by Theorems-1 and 2, and by the data obtained by using formulas (3-10), including the data given in the Appendix.

(2) The fact that the equation of Fermat's Grand Theorem has many solutions is proved by Theorems 3, 4, 5 and 6, and by the data obtained by using formulas (11-27).

3. The value of an irrational number, which is used in this article as a $\sqrt[3]{2}$ in the Electronic Encyclopedia of Integer Sequences, is given as equal: $1{,}259921049894873164767210607278228350570251 4\ldots$

However, installed in my laptops and computers, on the same machinery of my friends and family, the constantly updated Microsoft Excel rounds the values of this irrational number. Specifically, to a value where: $\sqrt[3]{2} \cong 1{,}25992104989487$. Therefore, when this number is raised back to degree 3, for example, in a Python program, the result appears: $(1{,}25992104989487)^3 = 1{,}9999999999999853 < 2$**,** that is, a very small but still an error equal to 1.4654943925025052066E-14. But actually, this shouldn't be the case. Because when you raise an irrational number to the 3rd degree. $\sqrt[3]{2}$ that is $\left(\sqrt[3]{2}\right)^3$, same Microsoft Excel, Python, or other more precise programs, gives a result exactly equal to the number 2, that is $\left(\sqrt[3]{2}\right)^3 = 2$.

A similar state of affairs is characteristic of calculations related to the number "pi" (π) with 100 trillion decimal places as of June 2022.

But Microsoft Excel gives these or similar errors in other cases and circumstances as well. Therefore, when checking the results of the author of the article in other, more accurate computers, when such or similar errors are detected, it is necessary to treat them not as errors of its author, who has repeatedly checked and rechecked them, but as errors of the program used, i.e. Microsoft Excel.

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8. Appendices

Positive integers found by solving the Beale equation Brute force

№	$A^x + B^y = C^z$	CPD $A, B \wedge C$	A^x	B^y	C^z
1	$4^3 + 2^6 = 2^7$	2	64	64	128
2	$4^6 + 2^{12} = 2^{13}$	2	4096	4096	8192
3	$2^{14} + 2^{14} = 2^{15}$	2	16384	16384	32768
4	**$2^{16} + 2^{16} = 2^{17}$**	**2**	**65536**	**65536**	**131072**
5	**$2^{16} + 4^8 = 2^{17}$**	**2**	**65536**	**65536**	**131072**
6	**$4^8 + 4^8 = 2^{17}$**	**2**	**65536**	**65536**	**131072**
7	$2^{18} + 2^{18} = 2^{19}$	2	262144	262144	524288
8	**$4^{10} + 16^5 = 8^7$**	2	**1048576**	**1048576**	**2097152**
9	**$16^5 + 4^{10} = 8^7$**	**2**	**1048576**	**1048576**	**2097152**
10	**$32^4 + 16^5 = 8^7$**	**2**	**1048576**	**1048576**	**2097152**
11	**$16^5 + 32^4 = 8^7$**	**2**	**1048576**	**1048576**	**2097152**
12	**$16^5 + 16^5 = 8^7$**	**2**	**1048576**	**1048576**	**2097152**
13	**$4^{10} + 16^5 = 2^{21}$**	**2**	**1048576**	**1048576**	**2097152**
14	**$16^5 + 4^{10} = 2^{21}$**	**2**	**1048576**	**1048576**	**2097152**
15	**$16^5 + 16^5 = 2^{21}$**	**2**	**1048576**	**1048576**	**2097152**
16	**$2^{20} + 2^{20} = 2^{21}$**	**2**	**1048576**	**1048576**	**2097152**
17	**$4^{10} + 2^{20} = 2^{21}$**	**2**	**1048576**	**1048576**	**2097152**
18	**$2^{20} + 4^{10} = 2^{21}$**	**2**	**1048576**	**1048576**	**2097152**
19	**$16^5 + 2^{20} = 2^{21}$**	**2**	**1048576**	**1048576**	**2097152**
20	**$2^{20} + 16^5 = 2^{21}$**	**2**	**1048576**	**1048576**	**2097152**
21	$4^{14} + 4^{14} = 2^{29}$	2	268435456	268435456	536870912
22	**$4^{20} + 4^{20} = 2^{41}$**	**2**	**1099511627776**	**1099511627776**	**2199023255552**
23	**$4^{20} + 16^{10} = 2^{41}$**	**2**	**1099511627776**	**1099511627776**	**2199023255552**
24	**$16^{10} + 4^{20} = 2^{41}$**	**2**	**1099511627776**	**1099511627776**	**2199023255552**
25	**$32^8 + 16^{10} = 2^{41}$**	**2**	**1099511627776**	**1099511627776**	**2199023255552**
26	**$16^{10} + 32^8 = 2^{41}$**	**2**	**1099511627776**	**1099511627776**	**2199023255552**
27	**$16^{10} + 16^{10} = 2^{41}$**	**2**	**1099511627776**	**1099511627776**	**2199023255552**
28	**$4^{20} + 16^{10} = 2^{41}$**	**2**	**1099511627776**	**1099511627776**	**2199023255552**
29	**$16^{10} + 4^{20} = 2^{41}$**	**2**	**1099511627776**	**1099511627776**	**2199023255552**
30	**$2^{40} + 2^{40} = 2^{41}$**	**2**	**1099511627776**	**1099511627776**	**2199023255552**
31	**$4^{20} + 2^{40} = 2^{41}$**	**2**	**1099511627776**	**1099511627776**	**2199023255552**
32	**$2^{40} + 4^{20} = 2^{41}$**	**2**	**1099511627776**	**1099511627776**	**2199023255552**
33	**$16^{10} + 2^{40} = 2^{41}$**	**2**	**1099511627776**	**1099511627776**	**2199023255552**
34	**$2^{40} + 16^{10} = 2^{41}$**	**2**	**1099511627776**	**1099511627776**	**2199023255552**
35	$27^3 + 54^3 = 3^{11}$	3	19683	157464	177147
36	$81^3 + 162^3 = 3^{14}$	3	531441	4251528	4782969
37	$243^3 + 486^3 = 3^{17}$	3	14348907	114791256	129140163
38	**$27^6 + 1458^3 = 81^5$**	**3**	**387420489**	**3099363912**	**3486784401**
39	**$27^6 + 1458^3 = 3^{20}$**	**3**	**387420489**	**3099363912**	**3486784401**
40	**$9^9 + 1458^3 = 9^{10}$**	**3**	**387420489**	**3099363912**	**3486784401**
41	**$3^{18} + 1458^3 = 81^5$**	**3**	**387420489**	**3099363912**	**3486784401**
42	**$729^3 + 1458^3 = 243^4$**	**3**	**387420489**	**3099363912**	**3486784401**
43	$27^4 + 162^3 = 9^7$	3	531441	4251528	4782969
44	$3^9 + 54^3 = 3^{11}$	3	19683	157464	177147
45	$3^{12} + 162^3 = 3^{14}$	3	531441	4251528	4782969
46	$3^{15} + 486^3 = 3^{17}$	3	14348907	114791256	129140163
47	$3^{18} + 1458^3 = 3^{20}$	3	387420489	3099363912	3486784401